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Ion Acoustic Solitons in a Streaming Relativistic Plasma with Negative Ions

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We analyze the effect of a nonzero streaming velocity of both positive and negative ions on the formation of solitary waves in a relativistic plasma. The thermodynamic situation is considered to be isothermal. For various values of $u_{\alpha 0/C}$ and $n_{\alpha o}/n_{\beta o}$ we obtain the variations of the amplitude and the width of the solitary wave.

1. INTRODUCTION

One of the most important aspects of nonlinear plasma theory is the study of solitary waves. During the last few years numerous papers have appeared discussing the various situations that may occur in a plasma. But a new phenomenon which has become very important only very recently is that of soliton formation in a relativistic plasma. Initiation of such a study was done in the paper of Das and Paul (1985). Later workers introduced other effects, such as Landau damping (Roy Chowdhury *et al.*, 1988) and the two-temperature effect (Roy Chowdhury *et al.*, 1990); the importance of wave propagation in a relativistic plasma was recognized some time ago because of its relevance in astrophysical contexts (Tsytorich, 1974; Stenflo and Tsintsadze, 1979) and in laser plasma interactions (Kaw and Dawson, 1970; Sukla *et al.*, 1986). In addition, the study of plasmas in the presence of negative ions is very important for the explanation of many astrophysical and laboratory events (Das and Karmakar, 1990; Tagare and Reddy, 1987; Tagare, 1986).

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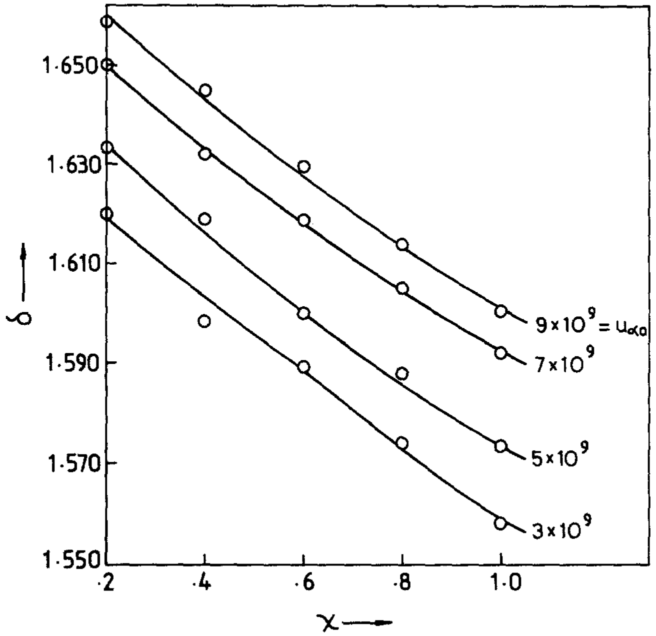


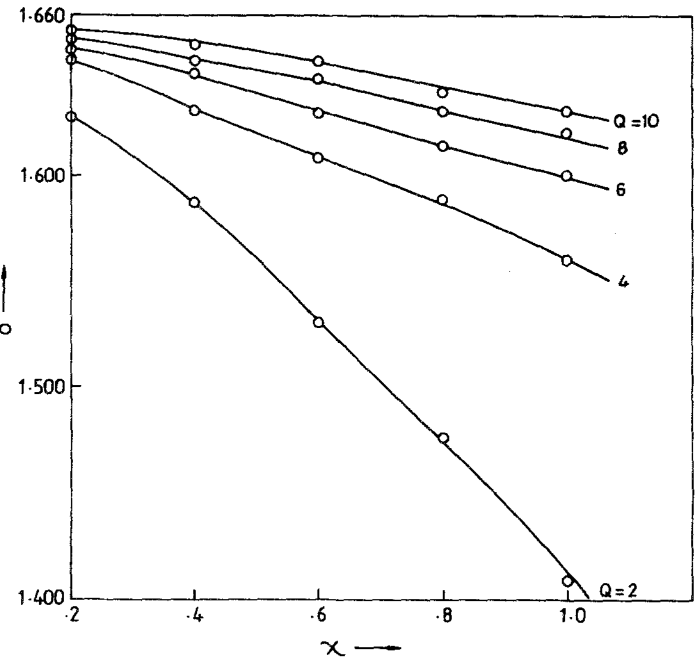
Fig. 1. (a) Plot of the width of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$, for fixed $Q=6$ and varying $u_{\alpha 0}$. (b) Plot of the width of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$, for fixed $u_{\alpha 0}=3\times 10^9$ and varying Q. (c) Plot of the width of the soliton against the ratio $u_{\alpha 0}/C$, for fixed $Q=6$ and varying $n_{\beta 0}/n_{\alpha 0}$.

In this paper we analyze a relativistic plasma containing two types of relativistic ions, one of which is negative, the mass ratios of the two ions being Q. Finally, we discuss the variation of the amplitude, the width of the solitary wave with respect to $u_{\alpha 0/C}$, $n_{\alpha 0}$, and $n_{\beta 0}$ (the equilibrium densities of the two ions), and $u_{\alpha 0}$, $u_{\beta 0}$ (the streaming velocities of the two types of ions).

2. FORMULATION

We consider an unmagnetized collisionless plasma consisting of isothermal electrons and two species of ions. For the propagation of small- but finite-amplitude waves in one dimension we assume the system to be weakly relativistic. The normalized plasma equations are

$$\left.\begin{aligned} \frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial x}(n_\alpha u_\alpha)&=0\\ \frac{\partial \bar{u}_\alpha}{\partial t}+u_\alpha\frac{\partial \bar{u}_\alpha}{\partial x}&=-\frac{\partial \phi}{\partial x}\end{aligned}\right\} \tag{1}$$



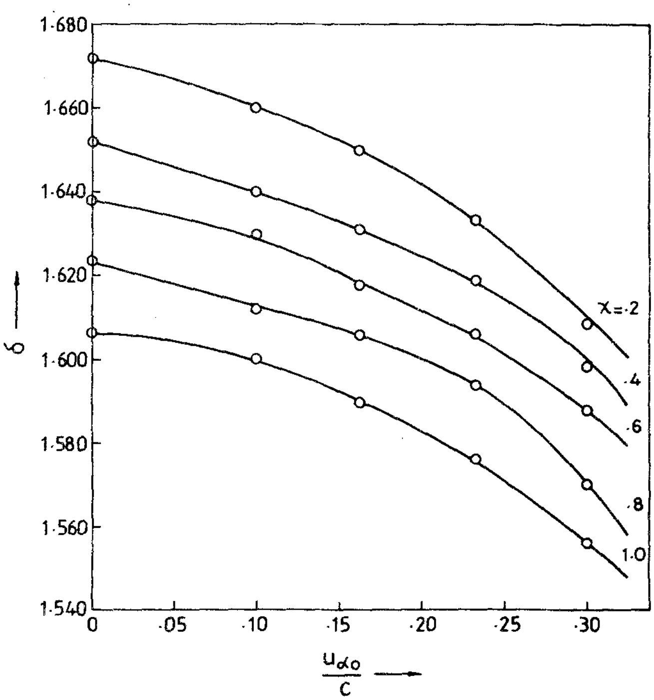


Fig. 1. Continued.

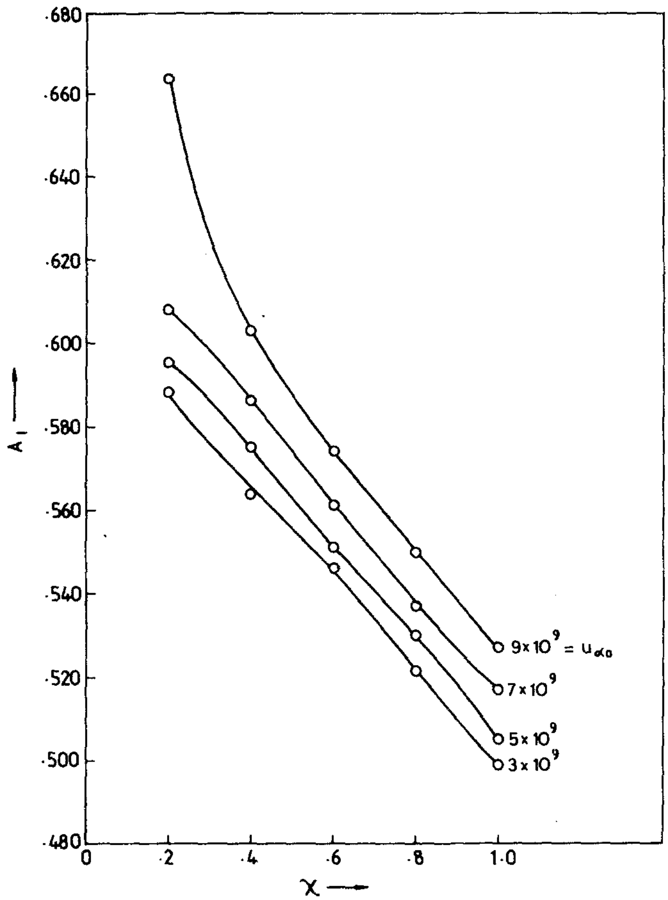


Fig. 2. (a) Plot of the amplitude of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$, for fixed $Q=6$ and varying $u_{\alpha 0}$. (b) Plot of the amplitude of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$, for fixed $u_{\alpha 0}=3\times10^9$ and varying Q. (c) Plot of the amplitude of the soliton against the ratio $u_{\alpha 0}/C$, for fixed $Q=6$ and varying $n_{\beta 0}/n_{\alpha 0}$.

$$\left.\begin{aligned}&\frac{\partial n_\beta}{\partial t}+\frac{\partial}{\partial x}(n_\beta u_\beta)=0\\&\frac{\partial \bar{u}_\beta}{\partial t}+u_\beta\frac{\partial \bar{u}_\beta}{\partial x}=-\frac{1}{Q}\frac{\partial \phi}{\partial x}\end{aligned}\right\}\tag{2}$$

$$\left.\begin{aligned}&\frac{\partial^2 \phi}{\partial x^2}=n_e+n_\beta-n_\alpha\\&n_e=e^{\phi}\end{aligned}\right\}\tag{3}$$

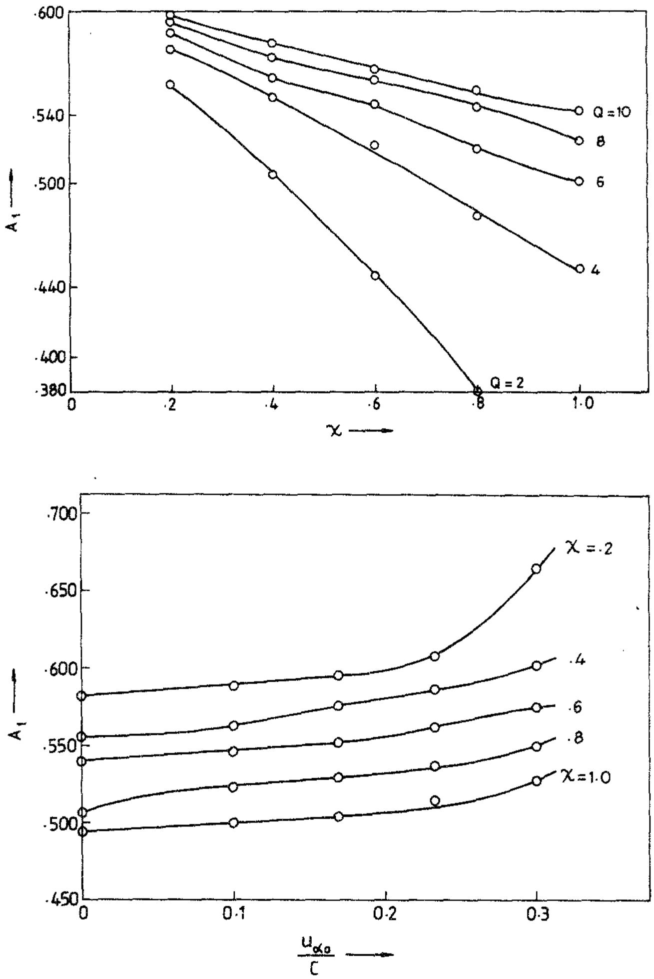


Fig. 2. Continued.

$$\bar{u}_\alpha \simeq u_\alpha\left(1+\frac{u_\alpha^2}{2c^2}\right)$$

$$\bar{u}_\beta \simeq u_\beta\left(1+\frac{u_\beta^2}{2c^2}\right) \tag{4}$$

n_α and n_β are the densities of the two types of ions and u_α and u_β are the corresponding velocities of positive and negative ions, respectively.

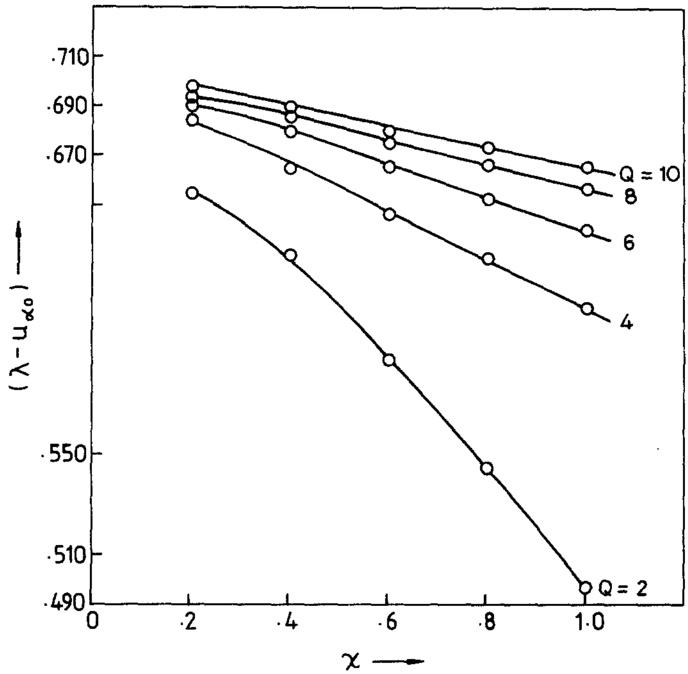


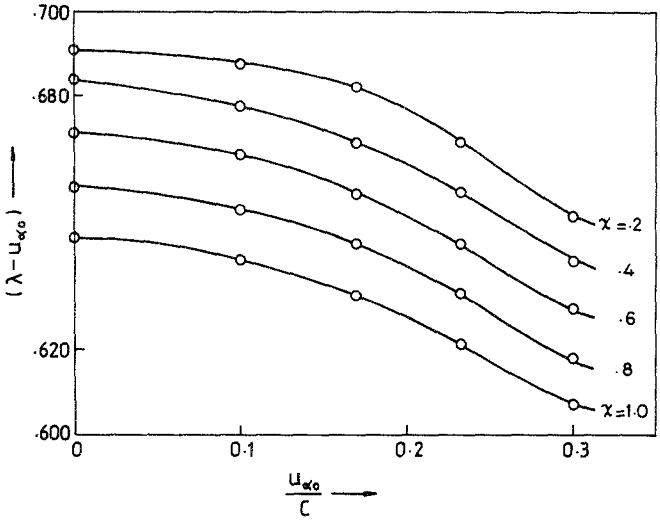
Fig. 3. (a) Plot of the difference of the phase velocity and $u_{\alpha 0}$ of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$ for fixed $u_{\alpha 0} = 3\times 10^9$ and varying Q. (b) Plot of the difference of the phase velocity and $u_{\alpha 0}$ of the soliton against the ratio $u_{\alpha 0}/C$ for fixed $Q = 6$ and varying $n_{\beta 0}/n_{\alpha 0}$. (c) Plot of the difference of the phase velocity and $u_{\alpha 0}$ of the soliton against the ratio $n_{\beta 0}/n_{\alpha 0}$ for fixed $Q = 6$ and varying $u_{\alpha 0}$.

We now set up a reductive perturbation-type procedure by defining

$$\begin{aligned} \xi &= \varepsilon^{1/2}(x - \lambda t) \\ \eta &= \varepsilon^{3/2} t \end{aligned} \tag{5}$$

along with the expansions

$$\begin{aligned} n_\alpha &= n_{\alpha 0} + \varepsilon n_{\alpha 1} + \varepsilon_2 n_{\alpha 2} + \cdots \\ n_\beta &= n_{\beta 0} + \varepsilon n_{\beta 1} + \varepsilon^2 n_{\beta 2} + \cdots \\ u_\alpha &= u_{\alpha 0} + \varepsilon u_{\alpha 1} + \varepsilon^2 u_{\alpha 2} + \cdots \\ u_\beta &= u_{\beta 0} + \varepsilon u_{\beta 1} + \varepsilon^2 u_{\beta 2} + \cdots \\ \phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \cdots \end{aligned} \tag{6}$$



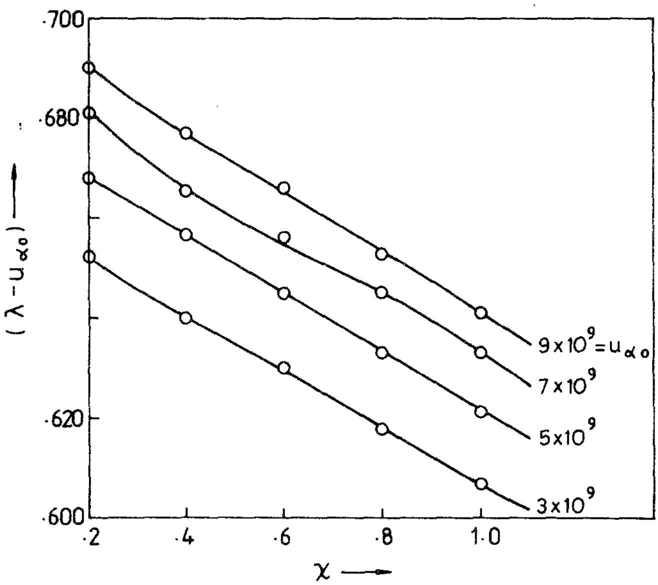


Fig. 3. Continued.

where ε is a small parameter and these expansions satisfy the normalized conditions

$$\begin{pmatrix} n_\alpha \\ n_\beta \\ u_\alpha \\ u_\beta \\ \phi \end{pmatrix} \rightarrow \begin{pmatrix} n_{\alpha 0} \\ n_{\beta 0} \\ u_{\alpha 0} \\ u_{\beta 0} \\ 0 \end{pmatrix} \quad \text{as} \quad x \rightarrow \pm\infty \tag{7}$$

Substituting (5) and (6) in (1)–(3) and collecting terms of different order of ε, we get, from first order of ε,

$$u_{\alpha 1} = \frac{\phi_1}{(\lambda - u_{\alpha 0})(1 + 3u_{\alpha 0}^2/2c^2)} \tag{8}$$

$$u_{\beta 1} = \frac{\phi_1}{Q(\lambda - u_{\beta 0})(1 + 3u_{\beta 0}^2/2c^2)} \tag{9}$$

$$\phi_1 = n_{\alpha 1} - n_{\beta 1}$$

$$n_{\alpha 1} = \frac{n_{\alpha 0} u_{\alpha 1}}{\lambda - u_{\alpha 0}} \tag{10}$$

$$n_{\beta 1} = \frac{n_{\beta 0} u_{\beta 1}}{\lambda - u_{\beta 0}} \tag{11}$$

From coefficients of ε^2 we have

$$\frac{\partial^3 \phi_1}{\partial \xi^3} - \phi_1 \frac{\partial \phi_1}{\partial \xi} - \left(\frac{\partial \phi_2}{\partial \xi} - \frac{\partial \eta_{\beta 2}}{\partial \xi} + \frac{\partial n_{\alpha 2}}{\partial \xi}\right) = 0$$

$$-(\lambda - u_{\alpha 0}) \frac{\partial n_{\alpha 2}}{\partial \xi} + \frac{\partial n_{\alpha 1}}{\partial \eta} + \frac{\partial}{\partial \xi}(n_{\alpha 1} u_{\alpha 1}) + n_{\alpha 0} \frac{\partial u_{\alpha 2}}{\partial \xi} = 0$$

$$\begin{aligned} &\left(1 + \frac{3u_{\alpha 0}^2}{2c^2}\right) \frac{\partial u_{\alpha 1}}{\partial \eta} - (\lambda - u_{\alpha 0})\left(1 + \frac{3u_{\alpha 0}^2}{2c^2}\right) \frac{\partial u_{\alpha 2}}{\partial \xi} \\ &\quad + u_{\alpha 1} \frac{\partial u_{\alpha 1}}{\partial \xi} + \frac{9u_{\alpha 0}^2}{2c^2} u_{\alpha 1} \frac{\partial u_{\alpha 1}}{\partial \xi} \\ &\quad - \frac{6\lambda u_{\alpha 0}}{2c^2} u_{\alpha 1} \frac{\partial}{\partial \xi} u_{\alpha 1} = -\frac{\partial \phi_2}{\partial \xi} \end{aligned} \tag{12}$$

with two more equations for $u_{\beta 2}$, $n_{\beta 2}$. Eliminating $u_{\alpha 2}$, $u_{\beta 2}$, $n_{\alpha 2}$, and $n_{\beta 2}$, we obtain the required KdV equation:

$$\frac{\partial \phi_1}{\partial \eta} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{13}$$

where A and B are given by

$$A = \frac{N}{D} \tag{14}$$

with

$$N = -1 - \frac{n_{\alpha 0} - n_{\beta 0}/Q^2}{(\lambda - u_{\alpha 0})^4 \gamma_\alpha^2}\left(2 \times \frac{1}{\gamma_\alpha} + \frac{9u_{\alpha 0}^2}{2c^2\gamma_\alpha} - \frac{3\lambda u_{\alpha 0}}{c^2 g_\alpha^3}\right)$$
$$D = \frac{-2}{(\lambda - u_{\alpha 0})^3 \gamma_\alpha}\left(n_{\alpha 0} - \frac{n_{\beta 0}}{Q}\right) \tag{15}$$

and

$$B = -\frac{1}{D}$$

3. SOLITON SOLUTION

A solitary wave solution of the KdV equation (13) can be easily obtained from the translation invariance and we write it as

$$\phi_1 = A_1 \operatorname{sech}^2(K\xi - \omega\eta) \tag{16}$$

where $A_1 = 3\omega/KA$ and $B = \omega/4K^3$, where A_1 is the amplitude of the wave and the width of the soliton is given as

$$\delta = 2\left(\frac{B}{U}\right)^{1/2} \qquad \text{where} \quad U = \frac{\omega}{K}$$

In Figures 1a–1c we plot the variation of the width of the soliton against, respectively: $n_{\beta 0}/n_{\alpha 0}$ for fixed Q; for fixed $u_{\alpha 0}$; and against $u_{\alpha 0}/C$ for fixed Q; while in Figures 2a–2c the same types of variation of the amplitude of the soliton are depicted again with respect to $n_{\beta 0}/n_{\alpha 0}$ and $u_{\alpha 0}/C$. Figures 3a–3c show similar variations of the phase velocity. It is now interesting to observe that such variations are very probable in two-ion plasmas consisting of He^+ in the presence of negative ions such as H^- or O_2^-. A similar situation may occur in a plasma of He^+ in the presence of N_2^-.

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